**An MSE-Based Image Similarity Measure: Performance over FFT-OFDM with QAM**

**Hadi J. K. Al-Yasiri، Zahir M. Hussain1,2**

**1 Faculty of Computer Science and Mathematics, University of Kufa**

**2 Professor (Adjunct), School of Engineering, Edith Cowan University, Australia**

**Abstract**

 In this work we present a simple image similarity measure based on relative mean squared error (MSE) after normalization. The performance of the proposed image similarity measure has been tested over FFt-OFDM system under the effect of Gaussian noise. A comparison with the well-known structural similarity measure (SSIM) has been made with different baseband modulation schemes: BPSK, QPSK, 8-PSK, and 16-QAM. It is shown that the proposed MSE measure, despite its simplicity, outperforms SSIM, especially at low SNR. The difference is more evident with high-order rectangular AM, where image similarity deteriorates under noise due to the increase in QAM symbol density.

**Keywords:** OFDM, FFT, QAM, image; digital communications

**1-Introduction:**

With increasing demand for high data rates in modern applications, OFDM is the main parallel transmission multi-carrier system which enables signals to be transmitted in parallel at different frequencies simultaneously from the same source [2]. Usually OFDM system use IFFT and FFT to multiplex the signals in parallel with reduced complexity algorithm at the transmitter and receiver respectively. The system employs guard interval or cyclic prefix (CP) so that the delay spread of the channel becomes longer than the channel impulse response; this is to minimize inter-symbol interference (ISI) between symbols [2,3,4].

Image transmission became important due to the increasing multi-media applications. However, noise can damage image features. In this work, we proposed an image quality index to test this damage over FFT-OFDM communication system systems.

**2- Background**

In this section we present an outline of FFT-OFDM system and structural similarity index (SSIM)

**FFT OFDM:**

In Fig. 1, the data $d\_{k}$ is processed by M-ary QAM baseband modulator to map the data before IFFT, with N subcarriers. Its output is the sum of the information signals in the discrete time as follows [2]:

$$x\_{k}=\frac{1}{N}\sum\_{m=0}^{N-1}X\_{m}e^{j2πkm/N}$$

where $x=\{x\_{k}|0\leq k\leq N-1\}$ is the signal represented by a sequence in the discrete time domain, $X=\{X\_{m}|0\leq m\leq N-1\}$ are complex numbers in discrete frequency domain representing DFT [1]. The cyclic prefix (CP) is added before transmission to minimize the inter-symbol interference. At the receiver side, the processed is reversed to obtain and decoded the data. The CP is removed to obtain the data in discrete time domain. The data is then processed to the Time-Domain (TD) windowing to eliminate narrowband interference before FFT.

At the receiver, FFT is performed on the received signal in discrete frequency domain as follows:

$$X\_{m}=\frac{1}{N}\sum\_{k=0}^{N-1}x\_{k}e^{-j2πkm/N}$$

See Fig.1 for details of the test system.



Fig.1: FFT-OFDM communication system.

**Image Structural Similarity Index (SSIM):**

In 2004, Wang et al. [5] introduced a new measure for image quality index named Structural Similarity index method **(**SSIM). There has been a wide use for this measure in image processing and communications.

The SSIM measure between two images X and Y is defined as follows:

$$SSIM(X,Y)=\frac{\left(2µ\_{x}µ\_{y}+C\_{1}\right)(2σ\_{xy}+C\_{2})}{\left(µ^{2}\_{x}+µ^{2}\_{y}+C\_{1}\right)(σ^{2}\_{x}+σ^{2}\_{y}+C\_{2})} $$

Where μx and μy represent the local means of images x and y, respectively, σx and σy represent the standard deviations, σxy is the cross-covariance of the two images،$σ^{2}\_{x}$ and $σ^{2}\_{y}$represent the variances, respectively, while the constants $C\_{1}$ and $C\_{2}$ are defined as$ C\_{1}=(K\_{1}L)^{2}$ and $C\_{2}=(K\_{2}L)^{2}$ with $K\_{1}=0.01,K\_{2}=0.03 $and L=255 [4]. SSIM ranges from 0 (no similarity) to 1 (full similarity).

**3. Proposed Similarity Index:**

In this work we propose a similarity measure ESIM based on normalized mean-squared error (MSE) between two M-by-N images X, Y as follows:

$$MSE\left(X,Y\right)=\frac{1}{M×N}\sum\_{i}^{}\sum\_{j}^{}[x\_{i,j}-y\_{i,j}]^{2}$$

$$ESIM\left(X,Y\right)=1-\frac{MSE(X,Y)}{max⁡[MSE\left(X,Y\right)]}$$

It will be shown that this measure ESIM outperforms the well-known SSIM under Gaussian noise for OFDM system communications.

**4. Implementation and Results**

 We simulated the FFT-OFDM system shown in Figure 1 and implemented similarity measures: the well-known SSIM and the proposed ESIM as per the equations above. In Figures (2-5), it can be seen that ESIM outperforms SSIM especially for higher order QAM, noting that higher-order QAM is important for multimedia communications.

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Fig.2.: Performance of proposed ESIM and SSIM under Gaussian noise with BPSK at baseband modulation.



Fig.3.: Performance of proposed ESIM and SSIM under Gaussian noise with QPSK at baseband modulation.



Fig.4.: Performance of proposed ESIM and SSIM under Gaussian noise with 8-PSK at baseband modulation.



Fig.5.: Performance of proposed ESIM and SSIM under Gaussian noise with 16-QAM at baseband modulation.

**4. Conclusion:**

 In this paper we proposed an efficient similarity measure for image transmission over FFT-OFDM system under Gaussian noise. The new measure is based on normalized mean-squared error. It is shown by numerical simulation that the proposed measure outperforms the well-known SSIM especially for higher order QAM modulation.

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